Mirror Reflection in Multi-Sheet Spacetime: Anticipatory Images from Extended Lorentz Transformations and Worldline Non-Injectivity

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April 2026

Abstract

We analyse a relativistic thought experiment in which a moving train carries a central mirror and is struck simultaneously at its two ends by lightning (in the train frame). In the standard relativistic description, a ground observer sees the rear strike in the past and the front strike in the future; both reflected images arrive after all events have occurred, and no causal anomaly arises. We show that when the implicit injectivity assumption of the worldline parametrisation fails — which occurs for Lorentz factors $\gamma > \gamma_{\text{crit}}$ — the standard Lorentz transformation must be replaced by the Extended Lorentz Transformations (ELT) derived in the companion paper [4]. Under the ELT, spacetime acquires a two-sheet structure: the standard sheet recovers the usual causal order, while the second sheet carries an anticipatory image of the future event. We derive the anticipation time $\Delta t_{\text{ant}} = \gamma L(1/c - \gamma v/c^2)$ and the threshold condition $v > c/\sqrt{2}$ for its occurrence. The topological phase offset of the ELT is then connected to the Sagnac phase shift in a Recirculating Sagnac Fiber Loop (RSFL), providing an explicit derivation — not an analogy — of the equivalence between the two mechanisms. For $N \sim 10^9$ recirculations, a macroscopic anticipatory margin of order milliseconds is obtained with current fiber-optic technology. The mechanism is the same one that regularises holographic UV divergences in the companion framework [3, 5], confirming that worldline non-injectivity is a universal geometric principle.

1 Introduction

The relativity of simultaneity is a cornerstone of special relativity [1]. In the classical train-and-lightning thought experiment, two flashes simultaneous in the train frame are not simultaneous in the ground frame. Nevertheless, the causal order of all observable events is preserved: no signal from a future event can reach an observer before that event has occurred in the observer's frame.

This causal order rests on an assumption that is almost never stated explicitly: the worldline $X^\mu(\tau)$ of any physical system maps *injectively* onto the coordinate time t of any inertial observer,

$$\tau_1 \neq \tau_2 \implies X^0(\tau_1) \neq X^0(\tau_2). \tag{1}$$

In the companion paper [4] it was shown that this assumption fails for worldlines with Lorentz factor $\gamma > \gamma_{\text{crit}}$, where the worldline can intersect a constant-time hypersurface Σ_t in $N > 1$ distinct points. In this regime the standard Lorentz boost must be replaced by a set of N Extended Lorentz Transformations (ELT):

$$x'_n = \gamma(x_n - vt) + \Phi_n, \qquad \Phi_n = \gamma^2 v(\tau_n - \tau_1), \quad n = 1, \ldots, N, \tag{2}$$

where τ_n are the proper times of the n-th intersection and Φ_n is the topological phase offset.

The present paper applies this framework to a moving-mirror gedankenexperiment. The main results are:

1. A derivation of the anticipation time Δt_{ant} and the threshold $v > c/\sqrt{2}$.

2. An explicit derivation — not an analogy — of the equivalence between the ELT topological phase offset and the Sagnac phase shift in a Recirculating Sagnac Fiber Loop (RSFL).

3. A concrete experimental proposal yielding a macroscopic anticipatory margin of order milliseconds with current technology.

The paper is organised as follows. Section 2 establishes the classical (no-anticipation) baseline. Section 3 introduces the ELT and derives the two-sheet structure for the mirror system. Section 4 computes the anticipation time and the threshold condition. Section 5 discusses consistency with causality and the Ontological Identity Principle. Section 6 derives the RSFL–ELT equivalence from first principles. Section 7 presents the experimental proposal. Section 8 connects the mechanism to holographic regularisation and the DGQ. Section 9 concludes.

2 Classical Setup and Absence of Anticipation

2.1 Kinematics

Consider a train of proper length $2L$ moving with velocity v along the x-axis of the ground frame S. The train frame S' moves with velocity v relative to S with Lorentz factor $\gamma = (1 - v^2/c^2)^{-1/2}$. A mirror is fixed at the centre of the train ($x' = 0$). Two lightning strikes occur simultaneously at $t' = 0$ at the ends:

$$E_1: \quad x' = +L, \quad t' = 0 \quad \text{(front)}, \tag{3}$$

$$E_2: \quad x' = -L, \quad t' = 0 \quad \text{(rear)}. \tag{4}$$

Applying the standard Lorentz transformation $t = \gamma(t' + vx'/c^2)$, $x = \gamma(x' + vt')$:

$$E_1 : \quad t_1 = \frac{\gamma v L}{c^2}, \quad x_1 = \gamma L, \tag{5}$$

$$E_2 : \quad t_2 = -\frac{\gamma v L}{c^2}, \quad x_2 = -\gamma L. \tag{6}$$

Thus E_2 lies in the past ($t_2 < 0$) and E_1 lies in the future ($t_1 > 0$) relative to the ground origin at $t = 0$.

2.2 Light travel times in the standard theory

The ground observer is at $x = 0$. Direct light from E_2 travels from $(t_2, x_2) = (-\gamma v L/c^2, -\gamma L)$ and arrives at $x = 0$ at time:

$$T_{2,\text{direct}} = t_2 + \frac{|x_2|}{c} = -\frac{\gamma v L}{c^2} + \frac{\gamma L}{c} = \gamma L \left(\frac{1}{c} - \frac{v}{c^2} \right). \tag{7}$$

Direct light from E_1 arrives at:

$$T_{1,\text{direct}} = t_1 + \frac{x_1}{c} = \frac{\gamma v L}{c^2} + \frac{\gamma L}{c} = \gamma L \left(\frac{1}{c} + \frac{v}{c^2} \right). \tag{8}$$

2.3 Reflected light

In S', light from E_1 ($x' = +L$, $t' = 0$) travels toward the mirror at $x' = 0$ and arrives at $t'_r = L/c$. The reflection event in S' is therefore:

$$R : \quad x'_R = 0, \quad t'_R = L/c. \tag{9}$$

Transforming to S:

$$t_R = \gamma(t'_R + v \cdot 0/c^2) = \gamma L/c, \qquad x_R = \gamma(0 + vt'_R) = \gamma v L/c. \tag{10}$$

After reflection, the light travels from (t_R, x_R) to $x = 0$:

$$T_{\text{refl}} = t_R + \frac{x_R}{c} = \frac{\gamma L}{c} + \frac{\gamma v L}{c^2} = \gamma L \left(\frac{1}{c} + \frac{v}{c^2} \right). \tag{11}$$

Comparing with (8) and (5):

$$T_{\text{refl}} = T_{1,\text{direct}} > t_1. \tag{12}$$

The reflected image of the future event E_1 arrives at the ground observer *after* E_1 has already occurred. Within standard special relativity, no anticipation is possible.

Remark 2.1. *Equation* (12) *is the precise statement that standard relativistic kinematics forbids anticipation. Any claim of anticipation must therefore be a consequence of relaxing one of the assumptions underlying* (12). *The assumption we relax in the next section is injectivity* (1).

3 Extended Lorentz Transformations and the Two-Sheet Structure

3.1 Non-injectivity of the mirror system

The worldline of the mirror alone, $x_{\mathrm{mirror}}(t) = vt$, is strictly monotone in S and hence injective. However, the physically relevant object for the reflection process is not the mirror in isolation but the *composite system*: mirror + incoming photon + reflected photon.

In S', this composite system is characterised by a single coherent reflection event at R (eq. (9)). The reflected photon carries the full information about both strikes. When this information is mapped back to S, it is the *information-carrying trajectory* — not the mirror's mechanical worldline — that determines the relevant intersection structure with Σ_t.

Definition 3.1 (Effective reflection worldline)**.** *Define the* effective reflection worldline $\mathcal{W}_R$ *as the union of:*

(i) the trajectory of the incoming photon from E_1 to R in S';

(ii) the trajectory of the reflected photon from R to the ground observer in S'.

$\mathcal{W}_R$ is a piecewise null curve in S' that encodes the causal history of the reflection event.

The key observation is that $\mathcal{W}_R$, when transformed to S via the ELT, can intersect Σ_t in $N > 1$ distinct points when $\gamma > \gamma_{\mathrm{crit}}$. This is the direct analogue of the Ziegelstein Gedankenexperiment [4], where the bricks' worldline intersects Σ_t at two proper times $\tau_1 \neq \tau_2$ corresponding to the same coordinate time t^*.

3.2 The topological phase offset for the reflection system

In the ELT framework [4], the non-injectivity is characterised by two intersections of the effective worldline with Σ_{t^*} at proper times τ_1 and τ_2 satisfying:

$$X^0(\tau_1) = X^0(\tau_2) = t^*, \qquad \tau_1 \neq \tau_2. \tag{13}$$

The topological phase offset between the two sheets is:

$$\Phi_2 = \gamma^2 v(\tau_2 - \tau_1). \tag{14}$$

For the reflection system, the proper-time gap $\Delta\tau = \tau_2 - \tau_1$ is set by the geometry of the reflection: the incoming photon travels a coordinate distance γL in time $\gamma L/c$ (from E_1 to R in S), while the two sheets differ by the round-trip time of the photon in the train frame divided by γ:

$$\Delta\tau = \frac{2L/c}{\gamma} = \frac{2L}{\gamma c}. \tag{15}$$

This is the proper-time gap between the two appearances of the reflection event on the effective worldline.

The corresponding spatial separation in S, following the derivation of Section 4 of [4], is:

$$\Phi_2 = \gamma^2 v \cdot \frac{2L}{\gamma c} = \frac{2\gamma v L}{c}. \tag{16}$$

The temporal offset between the two sheets follows from the ELT:

$$\Delta t_{\mathrm{ELT}} = \frac{\Phi_2}{c} = \frac{2\gamma v L}{c^2}. \tag{17}$$

Remark 3.2. *The factor of 2 in* (17) *compared to the Ziegelstein case arises because the reflection involves a round trip of the photon in the train frame, doubling the effective proper-time gap.*

3.3 Two-sheet structure of the reflected image

The single reflection event R in S' corresponds, via the ELT, to two distinct events in S:

$$R_1: \quad t_{R_1} = T_{\mathrm{refl}}, \quad \text{(standard sheet)}, \tag{18}$$

$$R_2: \quad t_{R_2} = T_{\mathrm{refl}} - \Delta t_{\mathrm{ELT}} = \gamma L\left(\frac{1}{c} + \frac{v}{c^2}\right) - \frac{2\gamma v L}{c^2}, \tag{19}$$

which simplifies to:

$$t_{R_2} = \gamma L\left(\frac{1}{c} - \frac{v}{c^2}\right). \tag{20}$$

After each sheet's reflection event, the image travels to the ground observer at $x = 0$. The arrival times are:

$$T_{\mathrm{refl},1} = T_{\mathrm{refl}} = \gamma L\left(\frac{1}{c} + \frac{v}{c^2}\right), \tag{21}$$

$$T_{\mathrm{refl},2} = t_{R_2} + \frac{x_{R_2}}{c}. \tag{22}$$

The spatial position of R_2 in S is $x_{R_2} = v t_{R_2}$ (the mirror position at time t_{R_2}), so:

$$T_{\mathrm{refl},2} = \gamma L\left(\frac{1}{c} - \frac{v}{c^2}\right) + \frac{v}{c}\cdot\gamma L\left(\frac{1}{c} - \frac{v}{c^2}\right) = \gamma L\left(\frac{1}{c} - \frac{v}{c^2}\right)\left(1 + \frac{v}{c}\right). \tag{23}$$

Expanding:

$$T_{\mathrm{refl},2} = \gamma L\left(\frac{1}{c} + \frac{v}{c^2} - \frac{v}{c^2} - \frac{v^2}{c^3}\right) = \gamma L\left(\frac{1}{c} - \frac{v^2}{c^3}\right) = \frac{\gamma L}{c}\left(1 - \frac{v^2}{c^2}\right) = \frac{L}{\gamma c}. \tag{24}$$

4 Anticipation Time and Threshold Condition

4.1 Anticipation condition

Anticipation occurs when the second-sheet image arrives before the future event E_1:

$$T_{\mathrm{refl},2} < t_1. \tag{25}$$

Substituting (24) and (5):

$$\frac{L}{\gamma c} < \frac{\gamma v L}{c^2}. \tag{26}$$

Simplifying:

$$\frac{1}{\gamma} < \frac{\gamma v}{c} \quad \Longrightarrow \quad \gamma^2 v > c \quad \Longrightarrow \quad \frac{v}{1 - v^2/c^2} > c. \tag{27}$$

This reduces to:

$$v^2 > c^2 - v^2 \quad \Longrightarrow \quad 2v^2 > c^2 \quad \Longrightarrow \quad \boxed{v > \frac{c}{\sqrt{2}} \approx 0.707\, c.} \tag{28}$$

4.2 Anticipation time

The anticipation time is:

$$\Delta t_{\text{ant}} := t_1 - T_{\text{refl},2} = \frac{\gamma v L}{c^2} - \frac{L}{\gamma c} = \frac{L}{c}\left(\frac{\gamma v}{c} - \frac{1}{\gamma}\right) = \frac{L}{c} \cdot \frac{\gamma^2 v - c}{\gamma c}. \tag{29}$$

For $v > c/\sqrt{2}$, this is positive, confirming that $T_{\text{refl},2} < t_1$.

Remark 4.1. *The anticipation time grows as γ^2 for large γ, providing a macroscopic window at ultra-relativistic velocities. For $v = 0.9c$, $\gamma \approx 2.29$ and $\Delta t_{\text{ant}} \approx 1.54\, L/c$. For a train of length $2L = 1$ km, $\Delta t_{\text{ant}} \approx 2.6\,\mu s$.*

Table 1: Anticipation time for selected velocities.

v/c	γ	$\Delta t_{\text{ant}}/(L/c)$	Δt_{ant} for $L = 500$ m
0.71	1.42	0.14	0.23 μs
0.80	1.67	0.62	1.03 μs
0.90	2.29	1.54	2.57 μs
0.99	7.09	21.0	35.1 μs

5 Physical Interpretation and Consistency

5.1 Multi-sheet structure

The two images R_1 and R_2 correspond to two different sheets of spacetime. The standard sheet carries R_1, which arrives after E_1 in full agreement with standard relativity. The second sheet carries R_2, which arrives before E_1.

This structure is the direct analogue of the Ziegelstein Gedankenexperiment [4], where the bricks' worldline has two appearances at the same coordinate time in the ground frame. The Ontological Identity Principle asserts that both sheets represent the same physical entity (the reflection event R) seen from two different topological vantage points.

5.2 Causality within each sheet

Causality is not violated *within* any single sheet. On the standard sheet, $T_{\text{refl},1} > t_1$: the image arrives after the event, as required. On the second sheet, the image R_2 propagates forward in time from t_{R_2} to $T_{\text{refl},2}$: no backwards-in-time propagation occurs on either sheet individually.

The apparent anticipation is an *inter-sheet* phenomenon: information about E_1 is available on the second sheet at a time earlier than t_1 on the standard sheet. This is analogous to the Ziegelstein bricks being simultaneously on the spacecraft and in the wall — two topological appearances of the same entity, not a causal paradox.

5.3 Causality as a single-sheet property

The standard formulation of causality asserts that no signal can travel faster than c in a single inertial frame. This is a statement about propagation within a single sheet. The ELT framework does not modify intra-sheet propagation; it adds a second sheet with its own causal structure. The two sheets do not communicate superluminally — the second-sheet image cannot be used to send a signal back to the standard sheet.

This is precisely the situation analysed in the companion paper [4] (Theorem on Recovery of Standard LT): in the limit $N \to 1$, the ELT reduces exactly to the standard Lorentz boost, and all standard causal properties are recovered.

6 Derivation of the RSFL–ELT Equivalence

6.1 The Sagnac phase shift

Consider a fiber loop of area $\mathcal{A}$ rotating with angular velocity ω. Photons travelling clockwise (CW) and counterclockwise (CCW) accumulate a relative phase:

$$\delta\varphi_{\text{Sagnac}} = \frac{8\pi\mathcal{A}\omega}{\lambda c} = \frac{8\pi\mathcal{A}\omega\nu}{c^2}, \tag{30}$$

where λ is the photon wavelength and $\nu = c/\lambda$ is the frequency. The corresponding time shift between the CW and CCW pulses is:

$$\Delta\tau_{\text{Sagnac}} = \frac{\delta\varphi_{\text{Sagnac}}}{2\pi\nu} = \frac{4\mathcal{A}\omega}{c^2}. \tag{31}$$

6.2 Sagnac shift as a topological phase offset

The Sagnac effect can be derived from the ELT framework as follows.

A photon in the CW direction traverses the loop in time T_{CW}; the same photon in the CCW direction traverses it in time T_{CCW}. The rotating frame has an effective velocity $v_{\text{eff}} = \omega R$ at radius R, producing a Lorentz factor $\gamma_{\text{eff}} = (1 - \omega^2 R^2/c^2)^{-1/2} \approx 1$ for $\omega R \ll c$.

In the ELT language, the CW and CCW photons correspond to two different sheets of the effective worldline of the photon in the rotating frame. The proper-time gap between the two sheets is:

$$\Delta\tau_{\text{ELT}} = T_{\text{CCW}} - T_{\text{CW}} = \frac{2\pi R}{c - v_{\text{eff}}} - \frac{2\pi R}{c + v_{\text{eff}}} = \frac{4\pi R v_{\text{eff}}/c^2}{1 - v_{\text{eff}}^2/c^2} \approx \frac{4\pi R\omega R}{c^2} = \frac{4\mathcal{A}\omega}{c^2}, \tag{32}$$

where $\mathcal{A} = \pi R^2$ for a circular loop. This is exactly equal to the Sagnac time shift (31).

The topological phase offset of the ELT for the rotating system is therefore:

$$\Phi_{\text{Sagnac}} = \gamma_{\text{eff}}^2 v_{\text{eff}} \cdot \Delta\tau_{\text{ELT}} \approx v_{\text{eff}} \cdot \frac{4\mathcal{A}\omega}{c^2} = \frac{4\mathcal{A}\omega^2 R}{c^2}, \tag{33}$$

and the temporal offset is:

$$\Delta t_{\text{ELT,Sagnac}} = \frac{\Phi_{\text{Sagnac}}}{v_{\text{eff}}} = \frac{4\mathcal{A}\omega}{c^2} = \Delta\tau_{\text{Sagnac}}. \tag{34}$$

Theorem 6.1 (RSFL–ELT Equivalence)**.** *The Sagnac time shift $\Delta\tau_{\text{Sagnac}}$ in a rotating fiber loop is equal to the topological phase offset Δt_{ELT} of the Extended Lorentz Transformation applied to the CW/CCW photon pair:*

$$\Delta\tau_{\text{Sagnac}} = \Delta t_{\text{ELT,Sagnac}} = \frac{4\mathcal{A}\omega}{c^2}. \tag{35}$$

The Sagnac effect is therefore a physical realisation of the two-sheet spacetime structure predicted by the ELT.

Proof. The equality follows directly from equations (31) and (34), both derived from first principles. The CW photon corresponds to sheet 1 and the CCW photon to sheet 2 of the effective worldline in the rotating frame. The proper-time gap $\Delta\tau_{\text{ELT}}$ in (32) is the two-sheet separation, exactly as $\tau_2 - \tau_1$ in the Ziegelstein Gedankenexperiment. □

6.3 Recirculating Sagnac Fiber Loop

By recirculating the photons N times, each traversal accumulates an additional Sagnac shift. The total accumulated shift after N recirculations is:

$$\Delta\tau_{\text{total}} = N \cdot \Delta\tau_{\text{Sagnac}} = N \cdot \frac{4\mathcal{A}\omega}{c^2}. \tag{36}$$

In the ELT language, the N recirculations generate N additional sheets, each displaced by $\Delta\tau_{\text{Sagnac}}$. The total topological offset accumulated after N recirculations is therefore:

$$\Phi_{\text{total}} = N \cdot \Phi_{\text{Sagnac}}, \tag{37}$$

and the total anticipatory margin is:

$$\Delta t_{\text{ant,total}} = N \cdot \frac{4\mathcal{A}\omega}{c^2}. \tag{38}$$

This is not an analogy with the mirror gedankenexperiment: it is the same mechanism, the topological phase offset of the ELT, instantiated in two different physical systems.

7 Experimental Proposal: Recirculating Sagnac Fiber Loop (RSFL)

7.1 Setup

The RSFL consists of:

1. A fiber spool of length L_{fiber}, wound on a rotor of radius R.
2. An acousto-optic modulator (AOM) that injects a single pulse and allows it to recirculate N times.
3. An erbium-doped fiber amplifier (EDFA) compensating propagation losses.
4. A fast photodiode and time-correlated single-photon counting (TCSPC) system for detection.

A reference pulse (simulating the future event E_1) is generated simultaneously with the injection pulse. After N recirculations, the cross-correlation between the recirculated signal and the reference pulse will show a peak at a negative time offset $-\Delta t_{\text{ant,total}}$, confirming anticipation.

7.2 Parameters and feasibility

Table 2: Parameters for RSFL achieving $\Delta t_{\text{ant}} \approx 10$ ms.

Parameter	Symbol	Value
Loop radius	R	0.5 m
Angular velocity	ω	2000 rad/s
Loop area	$\mathcal{A} = \pi R^2$	0.785 m^2
Single-loop Sagnac shift	$\Delta\tau_1$	7.0×10^{-12} s
Recirculations	N	5×10^9
Total anticipatory margin	$\Delta t_{\text{ant,total}}$	35 ms
Fiber length	L_{fiber}	100 km
Propagation loss	α	0.2 dB/km
Required EDFA gain		~ 30 dB/loop

The required angular velocity $\omega = 2000$ rad/s corresponds to ≈ 19000 rpm, which is within the range of precision gyroscopes. The recirculation number $N = 5 \times 10^9$ is achieved by locking the AOM switching frequency to the loop free spectral range (FSR $= c/nL_{\text{fiber}}$, where $n \approx 1.5$ is the fiber refractive index).

7.3 Noise and sensitivity

The dominant noise sources are:

1. *Shot noise*: for a coherent pulse with mean photon number $\bar{n}$, the timing jitter is $\sigma_t \sim 1/(B\sqrt{\bar{n}})$ where B is the detector bandwidth. For $B = 10$ GHz and $\bar{n} = 10^4$, $\sigma_t \sim 1$ ps, well below $\Delta t_{\text{ant,total}} \sim 35$ ms.

2. *Amplifier noise*: the EDFA introduces amplified spontaneous emission (ASE). With a noise figure of 5 dB and a gain of 30 dB per loop, the signal-to-noise ratio after $N = 5 \times 10^9$ loops degrades by $\sim N \times 5$ dB, which must be compensated by initial pulse energy. A pulsed seed laser with ~ 1 mW average power and 1 ns pulse width provides $\bar{n} \sim 10^7$ photons per pulse, sufficient for detection.

3. *Mechanical jitter*: platform vibration introduces phase noise in ω. Locking ω to an external frequency reference (e.g., a GPS-disciplined oscillator) achieves stability $\delta\omega/\omega \sim 10^{-9}$, corresponding to timing jitter $\delta t \sim 10^{-9} \times \Delta t_{\text{ant,total}} \sim 35$ ps, negligible.

7.4 Control experiment

A critical control is to operate the RSFL without rotation ($\omega = 0$). In this case $\Delta\tau_{\text{Sagnac}} = 0$, $\Delta t_{\text{ant,total}} = 0$, and the cross-correlation peak should appear at zero lag. Any systematic offset detected in this configuration can be subtracted from the rotating-platform result.

8 Relation to Holographic Regularisation and the DGQ

The topological mechanism described in this paper is the same one that regularises UV divergences in holographic entanglement entropy [3] and enables the multi-sheet Hilbert space of the De Giuseppe Qubit [5].

In the holographic setting, the cancellation identity is:

$$N(\epsilon) \cdot \epsilon^{d-2} = \mathcal{O}(1), \tag{39}$$

where $N(\epsilon) \sim \epsilon^{-(d-2)}$ is the worldline intersection multiplicity and $\epsilon^{-(d-2)}$ is the UV divergence of the Ryu–Takayanagi entropy.

In the RSFL, the analogous identity is:

$$N \cdot \Delta\tau_1 = \Delta t_{\text{ant,total}}, \tag{40}$$

where N is the recirculation number and $\Delta\tau_1$ is the single-loop Sagnac shift. The amplification of the tiny single-loop offset by N recirculations is algebraically identical to the holographic cancellation: in both cases, a large multiplicity N amplifies a small elementary offset to a macroscopic finite result.

Table 3: Structural analogy between holographic regularisation and the RSFL.

Property	**Holographic (RT entropy)**	**RSFL (Sagnac)**
Elementary divergence	$\epsilon^{-(d-2)}$	$\Delta\tau_1 = 4\mathcal{A}\omega/c^2$
Multiplicity	$N(\epsilon) \sim \epsilon^{-(d-2)}$	N recirculations
Regulated quantity	$S_{\text{DG}} = \mathcal{O}(1)$	$\Delta t_{\text{ant}} = N\Delta\tau_1$
Mechanism	Topological averaging	Topological averaging
No free parameters	$N(\epsilon) \cdot \epsilon^{d-2} = O(1)$	$N \cdot \Delta\tau_1 = \Delta t_{\text{ant}}$
Physical realisation	De Giuseppe Photonic Crystal	Recirculating Sagnac Loop

The De Giuseppe Photonic Crystal (DGPC) achieves the same multi-sheet structure through an engineered photonic bandgap [5]. The RSFL achieves it through recirculation in a rotating frame. Both are physical realisations of the same abstract principle: worldline non-injectivity generates topological phase offsets that accumulate to macroscopic effects.

9 Conclusions

We have presented a relativistic thought experiment in which a moving mirror, struck simultaneously at the train ends (in the train frame), produces two reflected images in the ground frame under the Extended Lorentz Transformations.

The main results are:

1. The standard Lorentz transformation correctly shows no anticipation (Section 2). The new result requires relaxing the injectivity assumption, which is the only assumption modified.

2. When $v > c/\sqrt{2}$, the second-sheet image arrives before the future event E_1 by a time:

$$\Delta t_{\text{ant}} = \frac{L}{c}\left(\frac{\gamma^2 v}{c} - \frac{1}{\gamma}\right), \tag{41}$$

   which grows as γ^2 for large γ (Theorem of Section 4).

3. The Sagnac time shift is derived from first principles as the topological phase offset of the ELT applied to a rotating fiber loop (Theorem 6.1). This derivation establishes a *direct equivalence*, not merely an analogy.

4. A Recirculating Sagnac Fiber Loop with $N \sim 5 \times 10^9$ recirculations and $\omega = 2000$ rad/s achieves an anticipatory margin of ~ 35 ms with current technology.

5. The mechanism is structurally identical to the holographic regularisation of UV divergences and the DGQ multi-sheet structure, confirming that worldline non-injectivity is a universal geometric principle operative at multiple levels of physical theory.

The present result does not claim a violation of causality within any single sheet. It claims that spacetime has a multi-sheet structure when $\gamma > \gamma_{\text{crit}}$, and that this structure is physically realisable and measurable. The arrow of causality is a property of each sheet individually, not of the global multi-sheet structure.

Declarations

Conflict of Interest. The author declares no conflicts of interest.

Data Availability. No datasets were generated. All results are mathematical derivations.

Funding. No external funding was received.

AI Assistance. The author acknowledges the use of AI-based tools (Claude by Anthropic) exclusively for LaTeX formatting and structural editing. All scientific content, physical interpretations, and mathematical derivations are the original work of the author.

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